

A Method to Estimate the Probability that any Individual Cloud-to-Ground Lightning Stroke was within any Radius of any Point

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A new technique has been developed to estimate the probability that a nearby cloud to ground lightning stroke was within a specified radius of any point of interest. This process uses the bivariate Gaussian distribution of probability density provided by the current lightning location error ellipse for the most likely location of a lightning stroke and integrates it to determine the probability that the stroke is inside any specified radius of any location, even if that location is not centered on or even with the location error ellipse. This technique is adapted from a method of calculating the probability of debris collision with spacecraft. Such a technique is important in spaceport processing activities because it allows engineers to quantify the risk of induced current damage to critical electronics due to nearby lightning strokes. This technique was tested extensively and is now in use by space launch organizations at Kennedy Space Center and Cape Canaveral Air Force Station. Future applications could include forensic meteorology.

Nomenclature

θ = angle between the collision plane coordinates and the rotated collision plane coordinates

ρ_{xz} = correlation coefficient of x and z

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σ_K = standard deviation of x coordinate of the diagonalized covariance ellipse in the rotated coordinate system
 σ_H = standard deviation of z coordinate of the diagonalized covariance ellipse in the rotated coordinate system
 σ_x = standard deviation of x
 σ_z = standard deviation of z
 σ_x = standard deviation of x in the rotated coordinate system
 σ = standard deviation of z in the rotated coordinate system
 μ_K = x coordinate of target circle in the (X Z) coordinate system
 μ_H = z coordinate of target circle in the (X Z) coordinate system
 A = collision cross sectional area (nautical miles² nmi²)
 $d\theta$ = the angle between two points on the target ellipse
 dH = integration step
 H = intermediate variable in the lightning probability algorithm
 P = probability
 pdf = probability distribution function
 r_A = radius of circle of cross sectional area A (nautical miles nmi)
 R = radius of ellipse (nmi)
 $R1$ = distance to first point on target ellipse (nmi)
 $R2$ = distance to second point on target ellipse (nmi)
 x = horizontal rectangular coordinate in the collision plane (nautical miles nmi)
 x = horizontal rectangular coordinate in the rotated collision plane (nmi)
 x = transformation variable that circularizes the x component of the probability ellipse (nmi)
 x_c = nominal distance of closed approach of two colliding objects (nmi)
 W = intermediate variable in the lightning probability algorithm
 z = vertical rectangular coordinate in the collision plane (nautical miles nmi)
 z = vertical rectangular coordinate in the collision plane (nautical miles nmi)

z = transformation variable that circularizes the z component of the probability ellipse (nm)

I Introduction

THE ability to accurately estimate the probability that an individual nearby cloud to ground lightning stroke was within a specified distance of any specified spaceport processing facility at Kennedy Space Center (KSC) or Cape Canaveral Air Force Station (CCAFS) is important to processing payloads and space launch vehicles before launch. Such estimates allow engineers to decide if inspection of electronics systems aboard satellite payloads, space launch vehicles, and ground support equipment is warranted due to induced currents from that stroke. If induced current damage has occurred, inspections of the electronics are critical to identify required fixes and avoid degraded performance or failure of the satellite or space launch vehicle. However, inspections are costly both financially and in terms of delayed processing for space launch activities. As such, it is important these inspections be avoided if not needed. At KSC/CCAFS, one of the main purposes of the Four Dimensional Lightning Surveillance System (4DLSS) [1-2] is detection of nearby strokes and determination of their peak current to support decisions to inspect electronics [3-5]. The high frequency of lightning occurrence in East Central Florida combined with the large amount of complex sensitive electronics in satellite payloads, space launch vehicles, and associated facilities makes those decisions critically important to space launch processing. The 4DLSS provides the data for 50th percentile location error ellipses for the best location for each stroke, which is then scaled to 95th or 99th percentile ellipses depending on customer requirements. This error ellipse is necessarily centered on the best location of the lightning stroke. The 4DLSS, however, has not been able to provide the probability of the stroke being within a customer specified distance of a point of interest. This paper presents a new method to convert the 4DLSS 50th percentile location error ellipse for best location of any stroke into the probability that the stroke was within any radius of any facility at CCAFS/KSC. This technique could be adapted for use with National Lightning Detection Network (NLDN) data. This new facility centric technique is a significant improvement over the stroke centric location error ellipses the 45th Weather Squadron (45WS) has provided in the past. This technique is adapted from a method of calculating the probability of debris collision with spacecraft [6-8].

II Methodology

A Background

In spacecraft collision probability and other applications at the instant of nominal closest approach the position uncertainty of the collision object relative to the asset being protected is described by a bivariate Gaussian probability density function (pdf) [6 8 9 10] as shown in the following equation

$$f(x, z) = \frac{1}{2\pi\sigma_x\sigma_z\sqrt{1-\rho_{xz}^2}} e^{-\left[\left(\frac{x}{\sigma_x}\right)^2 - 2\rho_{xz}\left(\frac{x}{\sigma_x}\right)\left(\frac{z}{\sigma_z}\right) + \left(\frac{z}{\sigma_z}\right)^2\right]/2(1-\rho_{xz}^2)} \quad (1)$$

where σ_x and σ_z = the standard deviations of x and z ρ_{xz} = correlation coefficient of x and z x and z are the designations for the rectangular coordinates in the collision plane

The probability of collision (Eq 2) is given by the two dimensional integral where A is the collision cross sectional area which is a circle with radius r_A [6]

$$P = \iint_A f(x, z) dx dz \quad (2)$$

There is no known analytical solution to the above integral when the two standard deviations σ_x and σ_z are not equal The solution is found by performing a numerical integration of the two dimensional Gaussian pdf [6 8 10]

The geometry used for spaceflight collision probability can also be used for estimation of the probability of an individual nearby lightning stroke contacting the surface within a specified distance of a specified point of interest as shown in Fig 1 Two methods of integrating the above probability were tested and gave identical results The first solution method was based on an algorithm by Patera [8] and the second solution method was based on an algorithm by Chan [11] Chan's algorithm ran much faster and therefore was selected as the algorithm for the 45WS lightning probability program

B The numerical integration technique [11]

This numerical integration technique is one in which the miss distance is given by a non central chi distribution with unequal variances [11]. The covariance matrix corresponding to the bivariate Gaussian pdf in Eq. 1 is [6]

$$C = \begin{bmatrix} \sigma_x^2 & \rho_{xz} \sigma_x \sigma_z \\ \rho_{xz} \sigma_x \sigma_z & \sigma_z^2 \end{bmatrix} \quad (3)$$

When the correlation coefficient ρ_{xz} is not zero there are undesirable off diagonal terms that overly complicate the calculation. In order to eliminate these terms the coordinate system (x, z) is rotated to a new coordinate system (x', z') such that the major and minor axes of the ellipse associated with the covariance are aligned along the coordinate axes and the new covariance matrix is [6]

$$C' = \begin{bmatrix} \sigma_{x'}^2 & 0 \\ 0 & \sigma_{z'}^2 \end{bmatrix} \quad (4)$$

The angle θ between the two coordinate systems is [6]

$$\theta = \frac{1}{2} \tan^{-1} \left[\frac{2\rho_{xz} \sigma_x \sigma_z}{(\sigma_x^2 - \sigma_z^2)} \right] \quad (5)$$

The KSC/CCAFS 4DLSS system does not provide the covariance matrix but instead provides the semi major axis, semi minor axis and the orientation of the semi major axis of the 50% location error ellipse relative to north. Therefore the angle θ in Eq. 5 is found using geometry where θ is the angle between the semi major axis of the lightning location uncertainty ellipse and line connecting the center of the lightning uncertainty ellipse and the center of the area of interest.

In the (x', z') system the Eq. 1 pdf becomes [6]

$$f(x', z') = \frac{1}{2\pi\sigma_{x'}\sigma_{z'}} e^{-\frac{1}{2} \left[\left(\frac{x'}{\sigma_{x'}} \right)^2 + \left(\frac{z'}{\sigma_{z'}} \right)^2 \right]} \quad (6)$$

and Eq. 2 the collision probability becomes [6]

$$P = \frac{1}{2\pi\sigma_x\sigma_z} \iint_A e^{-\frac{1}{2}\left[\left(\frac{x}{\sigma_x}\right)^2 + \left(\frac{z}{\sigma_z}\right)^2\right]} dx dz' \quad (7)$$

where

$$A' = A, r_{A'} = r_A, x'_p = x_e \cos \theta, z_p = x_e \sin \theta \quad (8)$$

For spacecraft collision x_e is the nominal distance of closest approach of the two colliding objects and (x_p, z_p) are the coordinates of the spacecraft relative to the debris. For lightning strike probability x_e (the distance between the position of the center of the strike location ellipse and the position of the target area) is calculated using the Haversine distance formula

The standard deviations in the new rotated coordinate system are calculated by dividing the semi major and semi minor axes of the 50% lightning positional confidence ellipse by the scaling constant used to scale standard error to the 50% confidence level. The scaling constant is

$$k = \sqrt{-2 * \ln(1 - 0.50)} \quad (9)$$

The probability is given by [11]

$$P = \frac{1}{2\sqrt{2\pi}\sigma_H} \int_0^{\sqrt{W}} \left[e^{-(H-\mu_H)^2/2\sigma_H^2} + e^{-(H+\mu_H)^2/2\sigma_H^2} \right] \left[\text{erf}(Z_1) + \text{erf}(Z_2) \right] dH \quad (13)$$

where

$$\begin{aligned} Z_1 &= \left[\sqrt{(W - H^2)} - \mu_K \right] / \sqrt{2}\sigma_K \\ Z_2 &= \left[\sqrt{(W - H^2)} + \mu_K \right] / \sqrt{2}\sigma_K \end{aligned} \quad (14)$$

The parameters μ_K and μ_H are the coordinates of the target circle in the (X, Z) coordinate system and σ_K and σ_H are the standard deviations of the diagonalized covariance ellipse shown in Eq. 4. The derivation of equations (13) and (14) above is shown in further detail in [11]. A detailed example of the calculations using a real world case is provided in Appendix A.

III Evaluation

The probability that any lightning strike is within any radius of any point of interest would be extremely difficult to estimate intuitively. As a result, given the high impact of the decisions on space launch operations, the tool developed for this application was extensively tested. Tests were conducted and are discussed in the following sections: 1) known mathematical solutions and 2) examination of real world events. The new technique passed all of the tests. Tests were also conducted to assure the probabilities calculated using the algorithm of Chan [11] matched probabilities calculated using the algorithm of Patera [8]. The probabilities calculated from the two algorithms were identical and thus are not shown here.

A Test Set 1

The first set of testing compared the lightning strike probability calculated using the 45WS lightning strike spreadsheet (which uses an adaptation of the numerical integration algorithm by [11]) to the corresponding circular probability from the CRC Handbook of Tables for Probability and Statistics [12]. Table 1 shows the probability from the new numerical integration technique for various inputs and the corresponding correct probability from the CRC Handbook. The values matched to within a tenth of a percent. These errors in the final digit may be due to round off error.

Table 1 Calculated probability vs CRC Handbook probability for various inputs

Semi major axis (nmi)	Semi minor axis (nmi)	Heading of semi major axis from true North	Point Of Interest latitude	Point Of Interest longitude	Strike Latitude	Strike Longitude	Radius around Point Of Interest (nmi)	Calculated probability	CRC Handbook probability [12]
3	3	15	28 6082	80 6041	28 6995	80 6041	3	0 095	0 095
3	3	15	28 6082	80 6041	28 631	80 6041	3	0 453	0 452
3	3	15	28 6082	80 6041	28 608	80 6041	3	0 500	0 499
1	1	15	28 6082	80 6041	28 608	80 6041	1	0 500	0 499
1	1	15	28 6082	80 6041	28 631	80 6041	1	0 200	0 200
1	1	15	28 6082	80 6041	28 6995	80 6041	1	0 000	0 000
1	1	15	28 6082	80 6041	28 608	80 6041	2	0 937	0 938

B Test Set 2

The second type of testing analyzed six real world lightning strikes near Space Launch Complex 39A on 3 August 2009. Figure 2 shows the spreadsheet used to generate the lightning report for those six strikes. Additional data on three of these six strikes are in Table 3. These strikes were selected because the closest point on the lightning position uncertainty ellipse was within 0.45 nautical miles of Launch Complex 39A, the key radius for assessing the need to inspect electronics for induced current damage to the Space Shuttle. Figures 9 through 11 are Google Maps depictions of three of these six strokes. The probabilities for a small area around a facility, even for a nearby stroke, may appear to be surprisingly low. For example, one strike just 0.65 nautical miles away (Figure 9) had only a 1.1% probability of being within the 0.45 nautical mile radius of Launch Complex 39A. All calculated probabilities are consistent with these real world events.

The KSC Electromagnetic Environmental Effects (EEE) Panel requested six more real world lightning strikes be investigated. These were recently investigated lightning strikes near Launch Complexes 39A or 39B where there was camera verification of the location of the strike. The EEE Panel wanted to compare the results of the new facility centric probabilistic technique to these cases where the true answers were known unambiguously. The data used for this analysis are in Table 4. Both 4DLSS and National Lightning Data Network (NLDN) cases were examined, depending upon which sensor system recorded the stroke. CGLSS strokes were obtained from 45WS 4DLSS. The NLDN usually provided flash data, so NLDN return stroke data were purchased as special StrikeNet⁴ reports from Vaisala Corporation. This was done to match the return strokes routinely provided by 4DLSS. Figures 12 through 14 show the probability results from these cases. As with the previous real world tests, all calculated probabilities were consistent with these additional real world events.

⁴ Vaisala, Inc. 2006. Vaisala StrikeNet Information. URL: http://www.vaisala.com/files/StrikeNet_Brochure.pdf

Table 2 Input values used for scenarios shown in Figures 9 through 11

Figure	Semi major axis of 50% confidence ellipse (km)	Semi major axis of 50% confidence ellipse (km)	Confidence	Heading (from true North) of semi major axis	Point of interest latitude (N)	Point of interest longitude (W)	Strike latitude (N)	Strike longitude (W)	Radius around point of interest (nmi)
9	0.4	0.2	0.99	300.7	28.60827	80.6041	28.6114	80.6113	0.45
10	0.3	0.2	0.99	293	28.60827	80.6041	28.6178	80.6069	0.45
11	0.2	0.1	0.99	20.3	28.60827	80.6041	28.5995	80.6113	0.45

IV Other Applications

The techniques and methods described in this paper clearly have application reaching far beyond the space program uses for which they were designed. The list of potential applications is many and varied and would be of interest to anyone seeking information pertaining to probability of lightning strike locations such as the power industry, aviation, or any industry sensitive to electrical overloads. This methodology is also applicable to forensic meteorology [13] where the question of whether lightning struck at or near a particular location is an issue in litigation. This technique could be applied to NLDN or any other cloud to ground lightning detection system that provides location error ellipses.

V Conclusion

A technique has been developed to calculate the probability that any nearby cloud to ground lightning stroke occurred within any radius of any point of interest. In practice, this provides the probability that a nearby lightning stroke was within a key distance of a facility rather than within the error ellipses centered on the stroke. This process uses the bivariate Gaussian distribution of probability density provided by the current lightning location error ellipse for the most likely location of a lightning stroke and integrates it to determine the probability that the stroke is inside any specified radius. This new facility centric technique which was tested extensively is much more useful to the space launch customers and has superseded the lightning error ellipse approach discussed in [3-4]. Potential other applications were briefly discussed in section IV.

Appendix

This appendix is an example of calculating the probability of any lightning stroke with a known error ellipse being within a circle of any radius around any point. It is provided to clarify the calculation process. An example calculation is shown in Table 4.

This example is a real world event from a lightning strike near the Space Shuttle launch pad 39A at 21:30 GMT on 14 Oct 2009 (ref Figure 12). Although the lightning data usually are from the cloud to ground component of the Four Dimensional Lightning Surveillance System (CG 4DLSS) (Murphy et al 2008, Roeder 2010) in this example a lightning stroke from the NLDN is used. We sometimes use StrikeNet reports that provide stroke data from the NLDN to double check the CG 4DLSS report.

- 1) Location of Launch Pad 39A: 28.60827486° N (or 0.499309 radians)
80.60411653° (ie 80.60411653° W or 1.406807 radians). This is also the center of the circle in which the lightning probability will be calculated.
- 2) Desired Radius For Probability of Lightning Around 39A: 0.45 nautical miles.
- 3) Lightning Stroke Data: Time/date: 0235 GMT, 16 August 2009; latitude: 28.6069° N (or 0.499285 radians); longitude: 80.6087° (ie 80.6087° W or 1.406887 radians); polarity/peak current: 48.0 kA; semi major axis of 50% confidence location ellipse: 0.6 km; semi minor axis of 50% confidence location ellipse: 0.4 km; orientation angle of location ellipse: 82° (clockwise from north).

Table 3 Lightning strike probability calculation process

Step	Action	Equation and other information	Calculation and Result
1	Convert semi major and semi minor axes from km to nmi	1 nmi = 1.852 km	0.6 km = 0.324 nmi semi major axis 0.4 km = 0.216 nmi semi minor axis
2	Calculate distance from lightning stroke to center of circle	Haversine Distance Formula Distance = Earth Radius * C <ul style="list-style-type: none"> Earth Radius = 3443.920086 nmi $C = 2 * \text{Atn2}[\sqrt{1-A}, \sqrt{A}]$ Atn2 is a two parameter arc tangent function which returns values in all four quadrants <ul style="list-style-type: none"> $A = \sin(dlat/2) * \sin(dlat/2) + \cos((\text{target lat})) * \cos((\text{stroke lat})) * \sin(dlon/2) * \sin(dlon/2)$ dlat = latitude difference from target to stroke = 	Distance = $3443.920086 * 7.4217 \times 10^{-5}$ = 0.2556 nmi $C = 2 * \text{Atn2}\{\text{sqr}(1.1377 \times 10^{-9}), \text{sqr}(1.577 \times 10^{-9})\}$ = 7.4217×10^{-5} $A = \sin(1.200 \times 10^{-5}/2) * \sin(1.200 \times 10^{-5}/2) * +$

	$28\ 60827^\circ\ 28\ 6069^\circ = 2\ 39959 \times 10^{-5}$ (radians) • dlon = longitude difference from circle to stroke = $80\ 60411^\circ\ 80\ 6085^\circ = 7\ 99967 \times 10^{-5}$ (radians)	$\cos(0\ 4993) * \cos(0\ 4993) * \sin(4\ 000 \times 10^{-5}/2) * \sin(4\ 000 \times 10^{-5}/2) = 1\ 3770 \times 10^{-9}$
3	Perform coordinate system rotation to eliminate the off diagonal term in the covariance matrix <ul style="list-style-type: none"> • $X = (\text{Longitude of Target} - \text{Longitude of Stroke}) * \cos(\text{Latitude of Strike})$ • $Z = \text{Latitude of Target} - \text{Latitude of Stroke}$ • $\theta = \alpha - ((\pi/2) - \text{Atn2}(X\ Z))$ • α is the orientation angle of the 50% lightning positional confidence ellipse • $X = \text{miss distance} * \cos(\theta \text{ (coordinate system rotation angle)})$ • $Z = \text{miss distance} * \sin(\theta)$ 	$X = (1\ 4068 - 1\ 4069) * \cos(0\ 4993) = 7\ 0231 \times 10^{-5}$ $Z = 0\ 49931 - 0\ 49929 = 2\ 400 \times 10^{-5}$ $0 = 1\ 431 - 1\ 571 - \text{Atn2}(7\ 023 \times 10^{-5}\ 2\ 400 \times 10^{-5}) = 0\ 1896$ $X = 0\ 2556 * \cos(0\ 1896) = 0\ 2510$ $Z = 0\ 2556 * \sin(0\ 1896) = 0\ 0482$
4	Calculate the standard deviations in the new rotated coordinate system <ul style="list-style-type: none"> • $\sigma_x = \text{Semi major axis of the 50\% lightning positional confidence ellipse / elliptical scaling constant used to scale standard error to the 50\% confidence level}$ • $\sigma_z = \text{Semi minor axis of the 50\% lightning positional confidence ellipse / elliptical scaling constant used to scale standard error to the 50\% confidence level}$ • Elliptical scaling constant k is $\sqrt{-2 * \ln(1 - \text{probability})}$ 	$k = \sqrt{-2 * \ln(1 - 0\ 50)} = 1\ 1774$ $\sigma_x = 0\ 3240 / 1\ 1774 = 0\ 2752$ $\sigma_z = 0\ 2160 / 1\ 1774 = 0\ 1834$
5	Calculate the probability that lightning stroke was within the target area of interest by performing a numerical integration using Simpson's rule of the lightning uncertainty ellipse over the area of the circle around the target of interest $P = \frac{1}{2\sqrt{2\pi}\sigma_H} \int_0^{\sqrt{W}} \left[e^{-(H-\mu)^2/2\sigma^2} + e^{-(H+\mu)^2/2\sigma^2} \right] [\text{erf}(Z_1) + \text{erf}(Z_2)] dH$ $Z_1 = \left[\sqrt{(W-H^2)} - \mu_K \right] / \sqrt{2}\sigma_K$ $Z_2 = \left[\sqrt{(W-H^2)} + \mu_K \right] / \sqrt{2}\sigma_K$ <ul style="list-style-type: none"> • μ_K and μ_H are the coordinates of the target circle in the (X Z) coordinate system • σ_K and σ_H are equal to σ_x and σ_z which are the standard deviations of the diagonalized covariance matrix • $W = \text{Radius around target}^2$ • $N = \text{the number of iterations to perform in the integration (for this example } N \text{ is set to } 200)$ • $DH = \sqrt{W} / N = \text{integration step}$ • B C and D are intermediate variables in the algorithm corresponding to various parts of the probability equation shown above <ul style="list-style-type: none"> • $B = \sqrt{2} * \sigma_x$ • $C = X / B$ • $D = 1 / (2 * \sqrt{2\pi} * \sigma_z)$ 	$W = \text{Radius around target}^2$ $W = 0\ 45^2 = 0\ 2025$ $DH = \sqrt{W} / N$ $DH = \sqrt{0\ 2025} / 200 = 0\ 00225$ $B = \sqrt{2} * \sigma_x$ $B = \sqrt{2} * 0\ 2752 = 0\ 3891$ $C = X / B$ $C = 0\ 2510 / 0\ 3891 = 0\ 6451$ $D = 1 / (2 * \sqrt{2\pi} * \sigma_z)$ $D = 1 / (2 * \sqrt{2\pi} * 0\ 1834) = 1\ 0874$ $H = \text{iteration no} * DH$

$$\bullet D = 1/(2 * \sqrt{2\pi} * \sigma_z)$$

$$H = 199 * 0.00225$$

$$H = 0.4478$$

- A, H, z1, z2, E, Γ , Erf(z1), Erf(z2), Q and sum are intermediate variables in the algorithm corresponding to various parts of the probability equation shown above

$$A = \sqrt{W - H^2}$$

$$A = \sqrt{0.2025 - 0.44775^2}$$

$$A = 4.494 \times 10^{-7}$$

- A loop is performed for j = 1 to 199. This example is shown for j = 199

$$z1 = A / B - C$$

$$z1 = 4.494 \times 10^{-2} / 0.3891 - 0.6451$$

$$z1 = 0.5296$$

$$z2 = A / B + C$$

$$z2 = 4.494 \times 10^{-2} / 0.3891 + 0.6451$$

$$z2 = 0.7606$$

$$\bullet \text{Sum} = 0$$

$$\bullet \text{Begin Loop here } H = j * DH$$

$$\bullet A = \sqrt{W - H^2}$$

$$\bullet z1 = A/B - C$$

$$\bullet z2 = A/B + C$$

$$\bullet \text{Erf}(x) = \text{error function} = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt$$

$$\bullet E = (H - Z)^2 / (2 * \sigma_z^2)$$

$$\bullet F = (H + Z)^2 / (2 * \sigma_z^2)$$

$$\bullet Q(j) = (e^E + e^F) * (\text{Erf}(Z1) + \text{Erf}(Z2))$$

$$\bullet \text{sum} = \text{sum} + (3 - (1)^j) * Q(j)$$

$$\text{Erf}(Z1) =$$

$$\text{ErrorFunction}(z1)$$

$$\text{Erf}(0.5296) = 0.5461$$

$$\text{Erf}(Z2) =$$

$$\text{ErrorFunction}(z2)$$

$$\text{Erf}(0.7606) = 0.7179$$

$$E = (H - Z)^2 / (2 * \sigma_z^2)$$

$$E = (0.4478 - 0.0482)^2 / (2 * 0.1834^2)$$

$$E = 2.372$$

$$\Gamma = (H + Z)^2 / (2 * \sigma_z^2)$$

$$\Gamma = (0.4478 + 0.0482)^2 / (2 * 0.1834^2)$$

$$\Gamma = 3.654$$

$$Q(j) = (e^E + e^F) *$$

$$(\text{Erf}(Z1) + \text{Erf}(Z2))$$

$$Q(199) = (e^{2.372} + e^{3.654}) *$$

$$(0.5461 + 0.7179) =$$

$$2.0467 \times 10^{-2}$$

$$\text{sum} = \text{sum} + (3 - (1)^j) * Q(j)$$

$$Q(j)$$

$$\text{sum} = \text{sum} + (3 - (1)^{199})$$

$$* 2.047 \times 10^{-2}$$

$$\text{sum} = 844.8952$$

$$\text{End Loop}$$

$$\text{sum} = \text{sum} + Q(0) + Q(N)$$

$$\text{sum} = 844.8952 + 2.9361$$

$$+ 0 = 847.8317$$

$$\text{Probability} = D * \text{sum} *$$

$$DH / 3$$

$$\text{Probability} = 1.0874 *$$

$$847.8317 * 0.00225 / 3 =$$

$$0.6914$$

$$E = (H - Z)^2 / (2 * \sigma_z^2)$$

$$E = (0.4478 - 0.0482)^2 / (2 * 0.1834^2)$$

$$E = 2.372$$

$$\Gamma = (H + Z)^2 / (2 * \sigma_z^2)$$

$$\Gamma = (0.4478 + 0.0482)^2 / (2 * 0.1834^2)$$

$$\Gamma = 3.654$$

$$Q(j) = (e^{-\Gamma} + e^{-E}) * (Erfi(\sqrt{E}) + Lrf(\sqrt{\Gamma}))$$

$$Q(199) = (e^{-2.372} + e^{-3.654}) * (0.5461 + 0.7179) =$$

$$2.0467 \times 10^{-2}$$

$$\text{sum} = \text{sum} + (3 - j) * Q(j)$$

$$Q(j)$$

$$\text{sum} = \text{sum} + (3 - j)^{199} * 2.047 \times 10^{-2}$$

$$\text{sum} = 844.8952$$

End Loop

$$\text{sum} = \text{sum} + Q(0) + Q(N)$$

$$\text{sum} = 844.8952 + 2.9361$$

$$+ 0 = 847.8317$$

$$\text{Probability} = D * \text{sum} * DH / 3$$

$$DH / 3$$

$$\text{Probability} = 1.0874 * 847.8317 * 0.00225 / 3 =$$

$$0.6914$$

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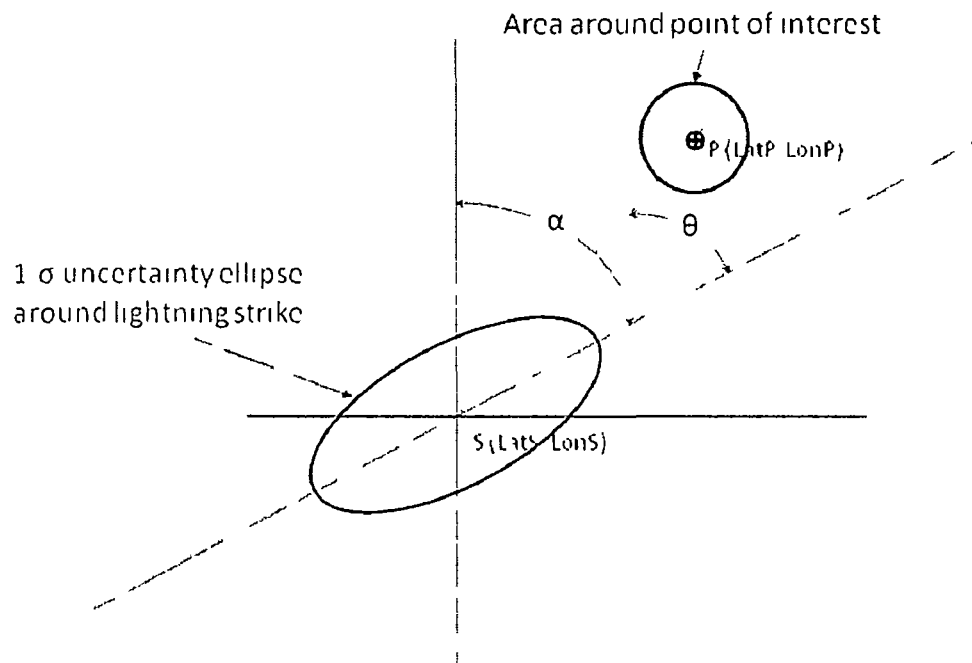


Fig 1 Schematic diagram of the angles used in probability calculation for a sample lightning location error ellipse α is the heading of the semi major axis of the lightning location uncertainty ellipse from true north θ is the angle between the semi major axis of the lightning location uncertainty ellipse and line connecting the center of the lightning uncertainty ellipse and the center of the area of interest

CG Lightning large Radii of interest correction - 25 Oct 10.xlsm

A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	
1	CX39A		Calculate								Open Map							
2																		
3	Latitude	28.60827486	User Def	28.602837	Current	50			27.5	29.50	Row Number	13495						
4	Longitude	-80.60411653	User Def	-80.6041	User-Defined	99			-81.5	-79.50	Perimeter P	20	Show Path					
5					Radius	0.45												
6	Date	Time	Azimuth Center	Range Center (nm)	Azimuth Closest	Range Closest (nm)	Magnitude (kAmps)	Target Inside Ellipse	Sensors in Solution	Probability (rad of interest)	Difference (nm)	CG Stroke Latitude	CG Stroke Longitude	Semi-Major Axis (nm)	Semi-Minor Axis (nm)	Orientation Angle	Closest Point Latitude	Closest Point Longitude
1948	08/03/2009	21:10:40.527	288	0.46	255	0.08	-39.8	No	3	46.072579%	0.38	28.6107	-80.6124	0.42	0.14	301.50	28.6079	-80.6056
1949	08/03/2009	21:10:40.601	296	0.42	080	0.12	-32.2	Yes	2	53.746968%	0.30	28.6114	-80.6113	0.56	0.28	300.70	28.6086	-80.6019
1951	08/03/2009	21:10:41.240	293	0.61	280	0.20	-48.1	No	4	12.147406%	0.41	28.6122	-80.6147	0.42	0.14	301.30	28.6089	-80.6078
2878	08/03/2009	21:23:04.062	346	0.59	357	0.27	-25.2	No	3	7.695824%	0.32	28.6178	-80.6069	0.42	0.28	293.00	28.6127	-80.6043
3160	08/03/2009	21:25:41.780	216	0.65	223	0.38	-24.6	No	5	1.125743%	0.27	28.5995	-80.6113	0.28	0.14	20.30	28.6036	-80.6090
3298	08/03/2009	21:27:18.421	222	0.56	236	0.32	-7.5	No	4	8.866220%	0.24	28.6014	-80.6112	0.28	0.14	12.90	28.6053	-80.6091
13494																		
13495																		
13496																		
13497																		

Fig. 2 Sample of lightning strikes where the closest point on the lightning position uncertainty ellipse was within 0.45 nmi of Launch Complex 39A on 3 August 2009.

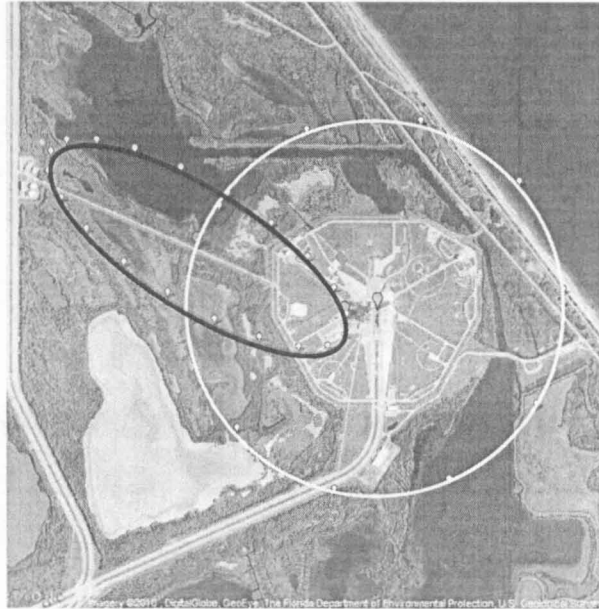


Fig. 3 Google Maps visualization of the 99% confidence uncertainty ellipse for one of the closest lightning strikes to Complex 39A on 03 August 2009. The black ellipse is the 99% lightning location uncertainty ellipse. The white circle is a 0.45 nmi radius around the point of interest. The center of the ellipse was within the 0.45 nmi radius. There is a 53.8% probability that the lightning occurred within that radius.

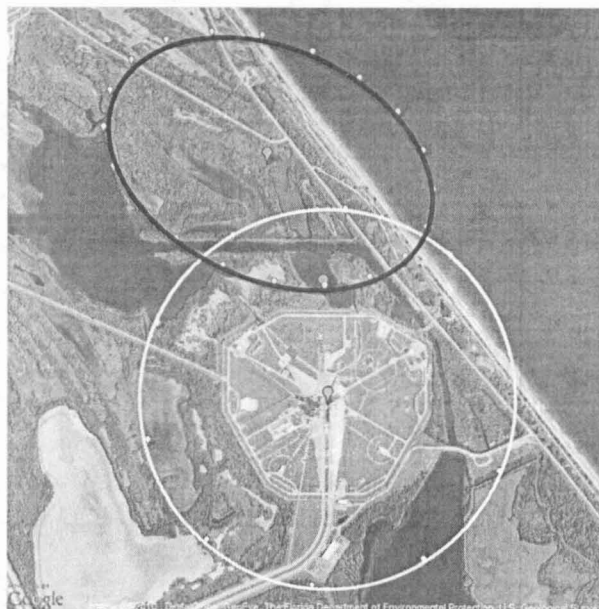


Fig. 4 Google Maps visualization of the 99% confidence uncertainty ellipse for a lightning strike near Complex 39A on 03 August 2009. The black ellipse is the 99% lightning location uncertainty ellipse. The white circle is a 0.45 nmi radius around the point of interest. This shows a probability of 7.7% of the lightning strike occurring within the 0.45 nmi radius.

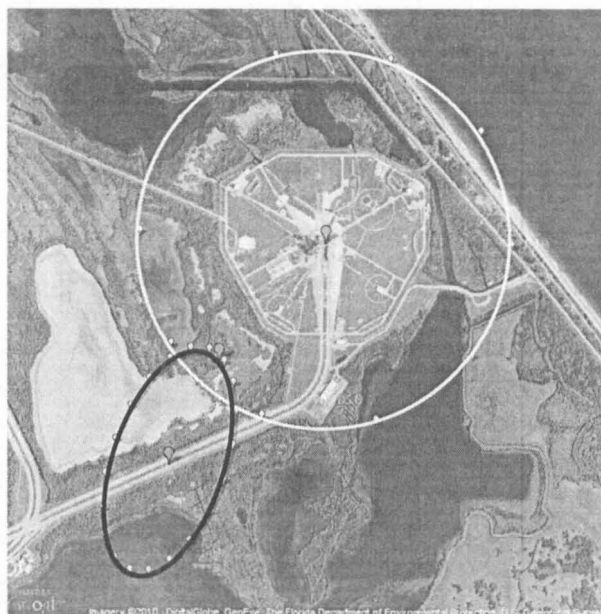


Fig. 5 Google Maps visualization of the 99% confidence uncertainty ellipse for nearby lightning strike to Complex 39A on 03 August 2009. The black ellipse is the 99% lightning location uncertainty ellipse. The white circle is a 0.45 nmi radius around the point of interest. Figure 11 shows a probability of 1.1% of the lightning strike occurring within the 0.45 nmi radius.



Fig. 6 Illustrates a probability of 69.1% of a lightning strike of amplitude -43.0 kA detected by NLDN occurring 0.26 nmi from the center of Launch Complex 39A on 8/16/2009. The black ellipse is the 99% lightning location uncertainty ellipse. The white circle is a 0.45 nmi radius around the point of interest.



Fig. 7 Illustrates a probability of 74.7% of a lightning strike of amplitude -71.4 kA detected by NLDN occurring 0.28 nautical miles from the center of Launch Complex 39A on 10/14/2009. The black circle is the 99% lightning location uncertainty ellipse. The white circle is a 0.45 nmi radius around the point of interest.

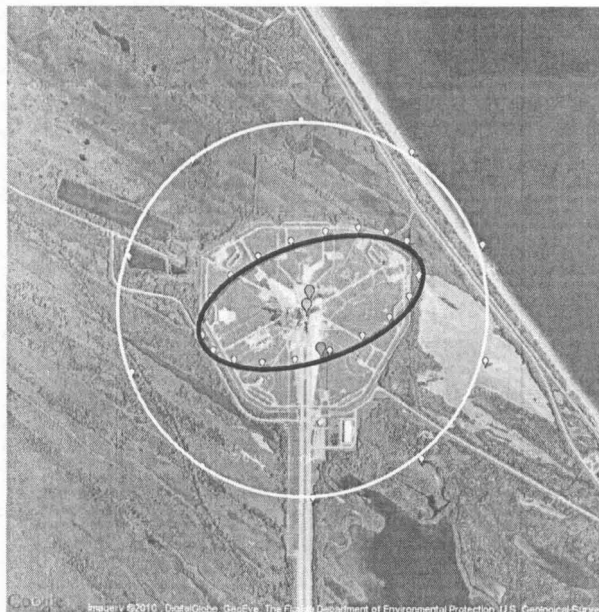


Fig. 8 Illustrates a probability of 99.9996% of a lightning strike of amplitude -21.7 kA detected by CGLSS occurring 0.04 nmi from the center of Launch Complex 39B on 6/27/2009. The black ellipse is the 99% lightning location uncertainty ellipse. The white circle is a 0.45 nmi radius around the point of interest.